**Assignment 4**

Question 2-

a.

Two generators G1 and G2​ are said to be equivalent if they produce the same sequence of values, in the same order.  
Formally:  
Let G1​ and G2​ be generator functions that yield infinite sequences (a1​,a2​,a3​,…) and (b1​,b2​,b3​,…), respectively.  
Then G1==G2​ **if and only if** for all i∈N we have: ai = bi.  
That is, for every position i, both generators yield the same value when advanced i times.

c.

We want to prove that the two generators Fib1 and Fib2 produce the same values at every index.  
In other words:

∀n∈N ≥ 1, Fib1[n] = Fib2[n]

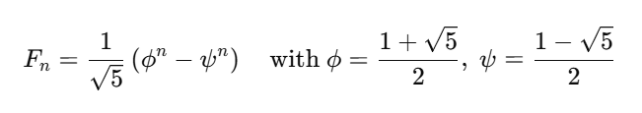
This follows from the fact that:

* The Fibonacci sequence defined by the recurrence:

F1 = F2 = 1, Fn = Fn−1 + Fn−2 for n ≥ 3.

is uniquely determined.

* Binet’s formula:



is a **closed-form expression** for the exact same Fibonacci numbers.

* It is a known and proven fact that Binet’s formula gives **integer** values for all n∈N, and satisfies the recurrence Fn = Fn−1 + Fn−2.

Therefore:

* Fib1 produces Fibonacci numbers using recurrence.
* Fib2 produces Fibonacci numbers using Binet’s formula.
* They yield the **same sequence**, hence:

Fib1 and Fib2 are equivalent generators.​

Question 3-

3.1.b

**Goal:** Prove that for all lists lst1, lst2 and continuation cont:

(append$ lst1 lst2 cont) = (cont (append lst1 lst2)

We do this by induction on the length of lst1, using applicative-order operational semantic.

**Base case:** lst1 = '() (n=0)

Then - (append$ ’() lst2 cont) => (cont lst2)

And - (append ’() lst2) => lst2 => (cont (append ’() lst2)) = (cont lst2)

So both expressions evaluate to the same result.

**Inductive step:**

Assume P(n) holds for lists of length n.  
Let lst1 = (cons x rest) be a list of length n+1, where rest has length n.  
We must show that:

append$ lst1 lst2 cont = cont (append lst1 lst2)

now:

(append$ lst1 lst2 cont)

→ (append$ rest lst2 (lambda (result) (cont (cons x result))))

By the **induction hypothesis** (since rest has length nnn):

append$ rest lst2 ( … ) = (lambda (result) … ) (append rest lst2)

So we get:

(cont (cons x (append rest lst2))) = (cont (append(cons x rest)lst2)) = (cont (append lst1 lst2))

Therefore P(n+1) holds.

**Conclusion:**

By induction on the length of lst1, we’ve proven that for all n ∈ N, for every list lst1 of length n, list lst2, and continuation cont:

**append$ lst1 lst2 cont = cont (append lst1 lst2)**